Optimized PMU placement by combining topological approach and system dynamics aspects

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Abstract—**In this paper a new Phasor Measurement Unit (PMU) placement methodology is proposed that combines approaches based on system dynamics and based on topological analysis. In this way, the deployment strategy of PMUs in a power system can be optimized. The method ensures the recognition of the most relevant inter-area oscillation modes from the beginning while reaching a fully observable system with a minimum number of PMUs in the long-term perspective. To achieve this, eigenvalue analysis is used to identify critical oscillation modes in the system and to find optimal PMU placements for their recognition. This result is then matched with full observability solutions obtained by integer linear programming. The methodology is demonstrated using the 39-bus New England Test System.**

Keywords-Optimal PMU Placement (OPP); Wide Area Monitoring; Eigenvalue Analysis; Integer Linear Programming; Full Observability

I. INTRODUCTION

PMUs measure the magnitude as well as the phase of voltages and currents in the network. They have various advantages as compared to classical remote terminal units (RTUs) that are usually used for Supervisory Control and Data Acquisition (SCADA) systems. The measurements of PMUs are time-stamped and can therefore be synchronized even if they are transmitted with different time delays. They also provide a much higher reporting rate. This enables practically real-time measurement and supervision of the system, which can be critical in increasingly dynamic power systems [1]. However, PMUs and their communication channels are costly, which is why an optimized PMU placement is needed [2].

Several publications have tried to classify the existing research on PMU placement approaches. In [3], a distinction is made in topology based and system aspects based algorithms. The former consider mainly the topology, i.e. which busbars are connected with each other by lines or transformers. The goal is to reach the full observability of the system with a minimum number of PMUs. Topology based algorithms typically formulate the Optimal PMU Placement (OPP) problem as a mathematical optimization problem. In [4], these are further classified into mathematical and heuristic algorithms. Short descriptions of different methods for each category are given. Integer linear programming (ILP), a mathematical algorithm, is explained and conducted in [5]. In [6] a method is proposed to calculate all equivalent solutions with the ILP approach.

System aspect based algorithms conduct a PMU placement with the goal to observe specific dynamic phenomena in the system. These algorithms require various dynamic system studies. In [7] an approach to divide all generators into groups and to select one representative generator per group to be monitored with a PMU is proposed.

In [8] three important aspects that are generally considered in combination with PMU placement for full observability are mentioned:

- Consideration of zero injection busses (see [5])
- Multistage implementation of PMUs (see [3])
- Redundancy in case of communication line/PMU outage (see [2])

The method proposed in this paper combines the advantages of topology and system aspects based algorithms for PMU placement. On the one hand, integer linear programming is used to calculate all possible solutions. On the other hand, the required PMUs to recognize critical power swings are determined using eigenvalue analysis. In the end the PMU placement solution with the highest overlap between these two approaches is selected. This combined approach is very flexible since the method to calculate the full observability solutions can be adjusted for example to consider aspect 1 or 3. Also the PMU locations for observation of dynamic processes in the system can be adjusted. Having an optimal PMU placement solution in mind from the beginning, cost savings can be reached by prevention of redundant installation of PMUs.

The system dynamics approach is discussed in chapter II. The basics of full observability and integer linear programming are explained in chapter III, followed by the presentation of the combination approach of both PMU placements. Finally, the method is verified in a study case in chapter V.

II. EIGENVALUE ANALYSIS FOR GENERATOR GROUPING

A. Mathematical principles

Eigenvalue analysis provides a much more systematic and efficient tool for the analysis of dynamic power system behavior than simulating disturbances in the time domain while observing the system reaction. The system is expressed as a linearized state matrix in the frequency domain. Its eigenvalues describe the possible power oscillation modes in the system, i.e. their frequency and their damping. The corresponding right eigenvectors give information about the observability of each mode. They can be used to find out where in the system a mode can be best observed and also which generators swing synchronously or against each other. The left eigenvectors on the other hand signal the controllability of each mode. This is of interest if for example the location of a PSS or FACTS element should be determined [9] [10].

The relationship between the i^{th} eigenvalue λ_i , the state matrix A and the right eigenvector ϕ_i, when $\phi_i \neq 0$, is given by (1).

$$A\phi_i = \lambda_i \phi_i \tag{1}$$

For the left eigenvector ψ_i on the other hand, it holds:

$$\psi_i A = \lambda_i \psi_i \tag{2}$$

The eigenvalues can be real or complex. Real eigenvalues describe non-oscillatory modes, complex eigenvalues on the other hand represent oscillatory modes. Therefore, complex eigenvalues are of interest when looking for inter-area oscillations. The eigenvalue of mode *i* can be written as in (3).

$$\lambda_i = \sigma_i \pm j\omega_i \tag{3}$$

with σ_i damping of the oscillation,

ω_i frequency of the oscillation.

Modes can be divided into categories by means of their frequency. Modes with a frequency of less than 0,1 Hz are typically controller modes. Swing modes generally lie between 0,1 and 3 Hz with inter-area modes typically on the lower end of this range (0,1-0,8 Hz) and regional modes with higher frequencies (0,8-3 Hz). From the damping and frequency, the damping ratio of mode *i* can be calculated according to (4), where a damping ratio of less than 5 % is usually viewed as critical [9].

$$\zeta_i = \frac{-\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}} \tag{4}$$

B. Generator grouping

For reliable recognition of specific power swing modes, PMUs need to be placed at generators that swing against each other. While the magnitude of the right eigenvector signals the observability of a mode at a certain generator, its phase indicates the oscillation phase and therefore allows the identification of coherent generator groups that swing in phase with each other. Since all generators of a group behave similarly, it is sufficient to place only one PMU at a representative node of each group. A sensible criterion to select this representative node is to choose the generator of the group that has the best observability and hence a relatively high magnitude of the right eigenvector.

The generator grouping can be conducted in a step-by-step manner, analyzing all the critical modes and categorizing the generators in ever-smaller groups. After this process it is possible to identify a small number of PMU locations that are sufficient to observe the most critical power swings.

III. PMU PLACEMENT FOR FULL OBSERVABILITY

A. Full Observability

When placing a PMU at a certain busbar, the voltage phasor and all current phasors of outgoing lines can be directly measured. Additionally, the voltage phasors at all system nodes connected to the PMU busbar through lines and transformers can be calculated. If the voltage phasors at both ends of a line or transformer are known, then the current between the two busbars can be calculated. The ability to directly or indirectly measure the voltage phasor on every bus in the system and all line current phasors is called full observability and brings different benefits for the grid operation, for example as a requirement for basing a SCADA system on PMU measurements. It is therefore seen as the long-term aim of the PMU deployment strategy. [8]

B. Integer linear programming

The described PMU placement problem for full observability can be described as an integer linear programming optimization problem which is expressed mathematically as in (5). Herein, *x* is a vector whose entries x_i indicate whether or not a PMU is installed at bus *i* by taking the values 1 or 0 respectively. The vector intcon defines which variables x_i can only be binary, which is the case here for all x_i . *A* and *b* as well as *Aeq* and *beq* are used to define equality and inequality constraints. Finally, it is possible to define lower and an upper bounds for every variable x_i using *lb* and *ub* [11].

$$\min_x f^T x \text{ subject to } \begin{cases} x(\text{intcon}) \text{ are integers} \\ A \cdot x \geq b \\ Aeq \cdot x = beq \\ lb \leq x \leq ub \end{cases} \tag{5}$$

The inequality constraint $A \cdot x \geq b$ formulates the demand for full observability using the connectivity matrix *A*. In a *n*-bus system this matrix is of the form $n \times n$ [5].

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if bus } i \text{ and bus } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Assuming that bus 1 is directly connected to the busses 2, 3 and 4, the first line in $A \cdot x \geq b$ would result in (7), assuring that bus 1 is directly or at least indirectly observable.

$$x_1 + x_2 + x_3 + x_4 \geq 1 \tag{7}$$

Since all constraints can be formulated as shown in this example no equality constraints are needed for the calculation of a full observability solution. This means that the entries of Aeq and beq are 0. Because all variables x_i are binary, lower and upper limitations of 0 and 1 respectively are adequate.

C. Multiple solutions

Calculation of multiple solutions can be a more complex task, since tools like Matlab often stop after finding the first solution. To get all possible and equally optimized solutions it is then necessary to calculate again with stricter constraints.

One approach is to stepwise prohibit the placement of a PMU at busbars suggested by previous solutions. As this has to be done for every new solution that is found, a recursive algorithm is required to calculate the full set of solutions. To set this further constraint the equality constraints can be used. If no PMU should be placed at bus i then $Aeq_{i,i} = 1$ and $beq_i = 0$ fulfills this requirement. After every calculation of the recursive algorithm it is necessary to check if a new solution can be found with the additional constraints and if the number of required PMUs is equal to the minimum. As it is possible that the same solution is obtained multiple times, duplicates have to be filtered [6].

IV. COMBINATION OF APPROACHES

A. Multistage implementation

As it was mentioned before PMU placement in two steps is suggested. Since the number of PMUs needed to observe the system dynamics is much lower than for full observability solutions, it is suggested to start with them. But since the final placement solution is known already these PMUs will not turn out to be redundant.

B. Maximum overlap solution

The first approach to combine the PMU placement based on system aspects and on the topology (full observability) is to calculate the overlaps of both solutions in order to identify the solution with the highest percentage of overlap. In case that the required PMUs from the system dynamics approach are not contained in any solution of the topological approach, i.e. no overlaps of 100 % is obtained, it is possible to increase the number of PMUs by the missing required locations.

C. New calculation with further constraints

If the amount of additional PMUs is greater than one, a new run of the full observability calculation should be started with the constraint of placing PMUs at the required busbars. There is the possibility that a solution with a PMU number that is only slightly higher than the minimal number can be found to be the better solution. In this case the overall number of PMUs could be less than taking the minimal number and adding the required PMUs.

D. Soften the requirements on PMU location

If it is the overall goal to keep the number of PMUs to the absolute minimum for full observability, further system study is necessary. It has to be studied if PMUs can be installed on other busbars than those found in the system dynamics study while still being able to observe the power swings in the system to a satisfying degree.

V. STUDY CASE

To validate the presented concept the method is applied to the 39-bus New England Test System, shown in Fig. 1. The test system consists of 10 generators and 46 lines on a 345 kV voltage level and a 10 kV voltage level on the low voltage side of the machine transformers. Generator 10 represents the aggregation of the residual system connected to busbar 39 with its dynamic behavior.

A. Eigenvalue analysis for generator grouping

The test system is studied with the dynamic power system analysis tool PSS®NETOMAC. With the integrated eigenvalue analysis function (NEVA) the eigenvalues and eigenvectors of the test system are calculated.

In Fig. 2 all modes that have been calculated with NEVA are visualized in the complex S-plane. Modes with a damping ratio below 5 % (see dashed 5 % isoline) are marked in red. From the different oscillation frequencies, controller modes and swing modes can clearly be differentiated in the diagram. Since controller modes do not represent a risk of instability, only the swing modes are of interest for the dynamic supervision of a system. [9]

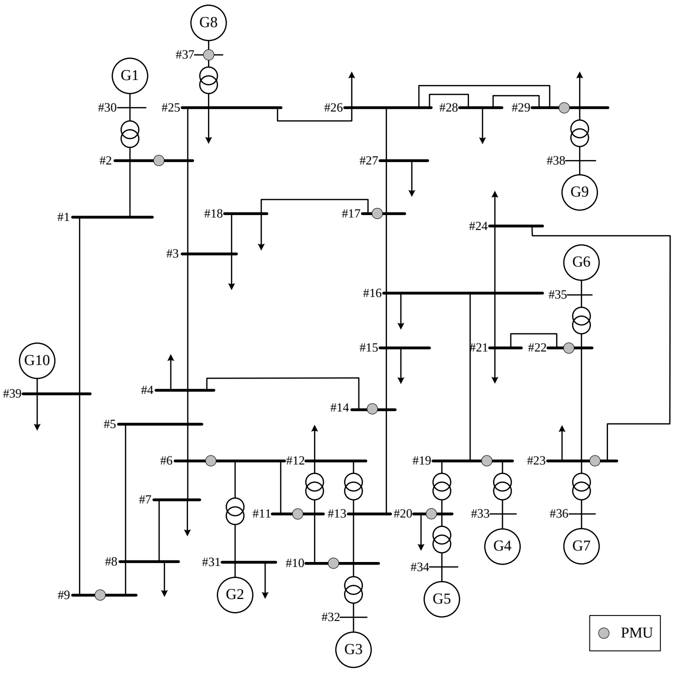

Fig. 1. 39-bus New England Test System with final PMU placement

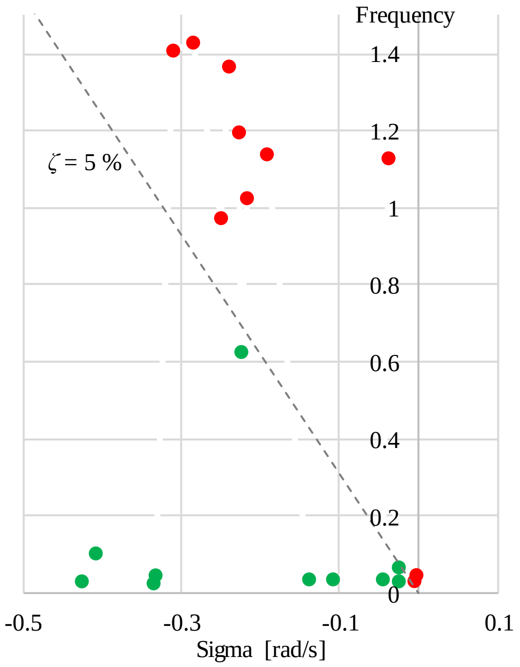

Fig. 2. Visualization of eigenvalues in complex S-plane

Table 1. Frequency and damping values of all modes

Mode type	*f* [Hz]	*ζ* [%]	*ω* [rad/sec]	*σ* [rad/sec]
Controller modes	0.029	88.236	0.18	-0.338
	0.031	2.436	0.197	-0.005
	0.033	12.669	0.209	-0.027
	0.034	89.23	0.216	-0.427
	0.037	51.297	0.232	-0.139
	0.038	19.164	0.236	-0.046
	0.039	41.297	0.245	-0.111
	0.048	1.24	0.302	-0.004
	0.048	73.911	0.305	-0.334
	0.069	6.431	0.435	-0.028
	0.105	52.629	0.662	-0.41
Swing modes	0.629	5.692	3.954	-0.225
	0.974	4.086	6.117	-0.25
	1.024	3.356	6.435	-0.216
	1.13	0.525	7.098	-0.037
	1.14	2.701	7.16	-0.193
	1.194	3.023	7.501	-0.227
	1.366	2.78	8.582	-0.239
	1.41	3.499	8.861	-0.31
	1.431	3.163	8.993	-0.285

In Table 1 the 20 modes that were calculated by eigenvalue analysis are listed, categorized in controller modes and swing modes.

Analysis of the right and left eigenvectors of the nine swing modes shows which generators participate in which mode and moreover which generator groups swing against each other. It turns out that there exists one mode (shaded in gray in Table 1) which shows a significant value of observability at all generators. Generators G1 to G9 all swing synchronously against G10. This is visualized in Fig. 3. The magnitude of the black fraction of the circles equivalents the magnitude of the respective right eigenvector entry of the specified generator. A quarter of a circle equivalents a magnitude of 1. The direction of the black segment represents the phase of the right eigenvector.

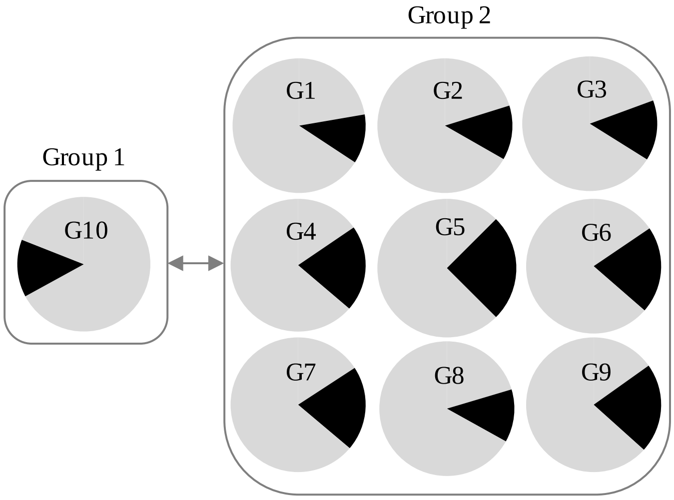

Fig. 3. Mode shape of most dominant mode

Supplementary simulations in the time-domain revealed that this mode is indeed the most dominant one which gets almost always excited when a disturbance occurs in the system. Therefore, observation of this power swing is found to be of primary importance in this power system.

This means that in both groups a representative generator for PMU placement has to be selected. Since Group 1 consists of only one Generator the first PMU has to be installed at Generator 10. From Group 2 Generator 5 and Generator 9 are selected as this is where the highest observability values are found for this mode. Since PMUs are regularly installed on the high-voltage side this means that PMUs have to be installed at busbars 20, 29 and 39.

B. PMU placement for full observability

With the PMU placement for the test system that was determined in the previous section it is possible to observe the most important power swing mode in the system. However, the system is not fully observable.

As it was described in section III, the full observability problem can be formulated as an integer linear programming optimization problem. The intlinprog function of Matlab was used for this paper to calculate the full observability solutions for the test system. The parameters of the function can be seen below. The inputs A and b have to be multiplied by negative one because the input format of intlinprog defines a turned inequality sign as compared to the equations in section III.

$$X = \mathrm{intlinprog}(f,\ \mathrm{intcon}, -A, -b, Aeq, beq, lb, ub) \qquad (8)$$

With this function only one possible solution is calculated. To calculate all solutions, the function has to be used multiple time with added constraints as has been explained in section III.

The minimum number of PMUs in the test system was determined to be 13. In total, 48 solutions for full observability with 13 PMUs were found (see Appendix, Table 5).

C. *Combination of approaches*

The results of the two approaches for the 39-bus test system are shown in Table 2.

Table 2. Results of both approaches

PMU placement acc. to	
Eigenvalue analysis	Full observability
PMUs at Busses 20, 29, 39	13 PMUs in total

A comparison of the PMU locations that have been obtained from the eigenvalue analysis with the 48 solutions for full observability has been conducted. It showed all 48 solutions have an overlap of at least 33,3 % because in every solution a PMU is installed at busbar 29. 24 of the solutions have a 66,6 % overlap with a PMU at busbar 20 as well. But no solution places a PMU at busbar 39.

Table 3. Example for overlap calculation

Solution no.	Full observability	Overlap [%]
1	2, 6, 9, 12, 14, 17, 22, 23, **29**, 32, 33, 34, 37	33,3 %
6	2, 6, 9, 10, 11, 14, 17, 19, **20**, 22, 23, **29**, 37	66,6 %

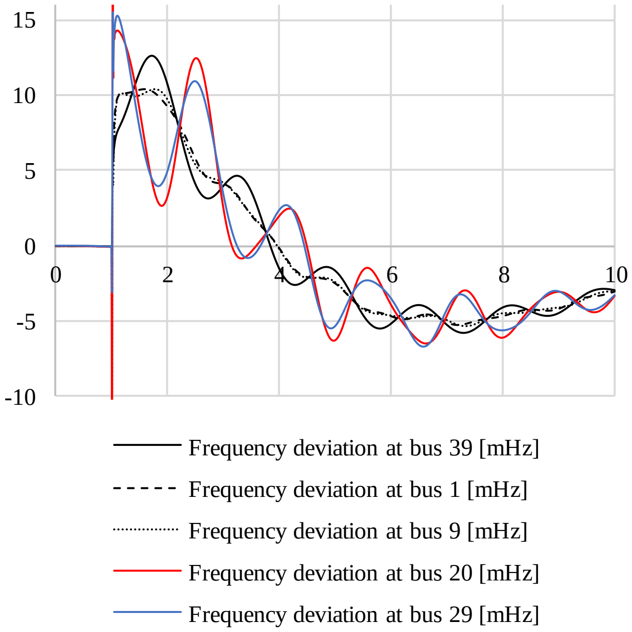

Fig. 4. Frequency at adjacent busses 1 and 9 during dominant mode

Since the number of additionally required PMUs is not greater than one, a new calculation would be redundant. Therefore, it is necessary to study if a PMU can be placed on a busbar other than busbar 39 while still obtaining the required information to observe the power swings in the system.

In Fig. 4 the frequency at bus 39 is compared to the two adjacent busses 1 and 9 during the dominant mode in a time-domain simulation. It can be seen that the frequency oscillation at those adjacent busses is much less pronounced than at bus 39.

However, the power swing recognition in established phasor data processing software is normally done using the voltage angle difference. From Fig. 5 it can be seen that the swing in the voltage angle difference is only slightly less pronounced when using adjacent busses, but still significant enough for successful swing recognition. It can also be seen that the signals measured at bus 1 and 9 are almost identical.

Checking the overlap with the full observability solutions again would suggest to place the PMU at bus 9 since it occurs in all solutions and bus 1 in none. Table 4 shows the overlap percentage for two example solutions. Consequently, solution number 6 is chosen as the final PMU placement. This placement is illustrated in Fig. 1.

Table 4. Second overlap calculation

Solution no.	Full observability	Overlap [%]
1	2, 6, **9**, 12, 14, 17, 22, 23, **29**, 32, 33, 34, 37	66,6 %
6	2, 6, **9**, 10, 11, 14, 17, 19, **20**, 22, 23, **29**, 37	100 %

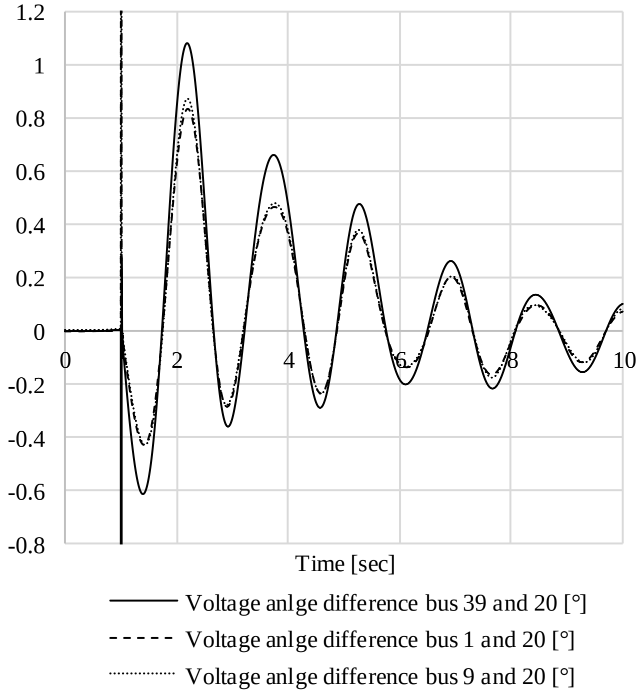

Fig. 5. Comparison of voltage angle difference between bus 39, 1 and 9 to bus 20 during dominant mode

VI. SUMMARY

With the increasing importance of Wide Area Measurement Systems (WAMS), cost effective and technically optimized PMU deployment strategies are needed. Several approaches for optimal PMU placement that serve different requirements exist. In this paper, a combination of two methods was proposed to harness the benefits of both. The aim was to find a small set of PMUs for monitoring the most critical power swing modes already from the beginning of PMU deployment and to combine it with a full observability solution as a long-term goal. This way, the SCADA systems can benefit from near real-time data from PMUs to a maximum degree and in an economical way. The suggested method was thoroughly discussed in theory and demonstrated on the 39-bus New England Test System.

APPENDIX

Table 5. Full observability solutions for New England Test System

No.	Busbar numbers with PMU												
1	2	6	9	12	14	17	22	23	29	32	33	34	37
2	2	6	9	13	14	17	22	23	29	32	33	34	37
3	2	6	9	11	14	17	22	23	29	32	33	34	37
4	2	6	9	10	11	14	17	22	23	29	33	34	37
5	2	6	9	10	11	14	17	19	22	23	29	34	37
6	2	6	9	10	11	14	17	19	20	22	23	29	37
7	2	6	9	10	11	14	17	19	20	22	23	25	29
8	2	6	9	10	11	14	17	19	22	23	25	29	34
9	2	6	9	10	11	14	17	20	22	23	29	33	37
10	2	6	9	10	11	14	17	20	22	23	25	29	33
11	2	6	9	10	11	14	17	22	23	25	29	33	34
12	2	6	9	11	14	17	19	22	23	29	32	34	37
13	2	6	9	11	14	17	19	20	22	23	29	32	37
14	2	6	9	11	14	17	19	20	22	23	25	29	32
15	2	6	9	11	14	17	19	22	23	25	29	32	34
16	2	6	9	11	14	17	20	22	23	29	32	33	37
17	2	6	9	11	14	17	20	22	23	25	29	32	33
18	2	6	9	11	14	17	22	23	25	29	32	33	34
19	2	6	9	10	13	14	17	22	23	29	33	34	37
20	2	6	9	10	13	14	17	19	22	23	29	34	37
21	2	6	9	10	13	14	17	19	20	22	23	29	37
22	2	6	9	10	13	14	17	19	20	22	23	25	29
23	2	6	9	10	13	14	17	19	22	23	25	29	34
24	2	6	9	10	13	14	17	20	22	23	29	33	37
25	2	6	9	10	13	14	17	20	22	23	25	29	33
26	2	6	9	10	13	14	17	22	23	25	29	33	34
27	2	6	9	13	14	17	19	22	23	29	32	34	37
28	2	6	9	13	14	17	19	20	22	23	29	32	37
29	2	6	9	13	14	17	19	20	22	23	25	29	32
30	2	6	9	13	14	17	19	22	23	25	29	32	34
31	2	6	9	13	14	17	20	22	23	29	32	33	37
32	2	6	9	13	14	17	20	22	23	25	29	32	33
33	2	6	9	13	14	17	22	23	25	29	32	33	34
34	2	6	9	10	12	14	17	22	23	29	33	34	37
35	2	6	9	10	12	14	17	19	22	23	29	34	37
36	2	6	9	10	12	14	17	19	20	22	23	29	37
37	2	6	9	10	12	14	17	19	20	22	23	25	29
38	2	6	9	10	12	14	17	19	22	23	25	29	34
39	2	6	9	10	12	14	17	20	22	23	29	33	37
40	2	6	9	10	12	14	17	20	22	23	25	29	33
41	2	6	9	10	12	14	17	22	23	25	29	33	34
42	2	6	9	12	14	17	19	22	23	29	32	34	37
43	2	6	9	12	14	17	19	20	22	23	29	32	37
44	2	6	9	12	14	17	19	20	22	23	25	29	32
45	2	6	9	12	14	17	19	22	23	25	29	32	34
46	2	6	9	12	14	17	20	22	23	29	32	33	37
47	2	6	9	12	14	17	20	22	23	25	29	32	33
48	2	6	9	12	14	17	22	23	25	29	32	33	34

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