System identification for dynamic equivalent model

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***Abstract*— The dynamic equivalent model is an effective approach to balance the efficiency and accuracy of simulation. Using operation point based estimation method, the elaborate power network could be reduced to a single transfer function. It can capture the steady state and transient behaviors of original grid with a high degree of accuracy. In this paper, a dynamic equivalent model of power network can be achieved by using the estimation based identification method, which will hold the dynamic response and the order of model can be reduced in simple form and good generality. This model maintains the physical meaning of the variety of the structure and can be more efficient in computation compared with the original model. This paper introduces three different estimation methods in system identification. The proposed methods can be applied in diverse sceneries, which is stipulated in normal operation.**

***Index Terms*—dynamic equivalent model, system identification, power network.**

I. Introduction

The reliable environment with a friendly power supply is one of the intense issues in the future. To achieve this goal, the German government has made a long-term plan. In 2050, the variable renewable energy(VRE) should reach around 80 percent for the power supply [1]. With the development of the grid interconnection, the scope of power network is expanding, in addition, the degree of complexity of grid is also increasing. It is becoming increasingly difficult to ignore the instability of dynamic state in the network stemmed from a mass of converter and inverter [2]. Specially, the widely used primary VRE are wind and solar energy, which are usually connected with the grid in middle voltage level and low voltage level. [3][4][5] With the increasing number of converter, the degree of intensity of them is getting higher, which can not only raise the potential of interaction between the converter, but also increase the instable rate of the power grid. For the aim of finding a solution to realize the analysis of the interaction between the different converters and the dynamic response, it is necessary to find out a reduced model of grid, which owns easier structure and can be analyzed easily.

The previous approach for finding a reduced equivalent model was first described by Ward in 1949. Ward examined circuits which had at least one generator and load while most buses were passive buses without load und generator connected to them. Over the last decades, attempts have been made to develop the methods, one of which is called REI. REI is an abbreviation for “Radial Equivalent Independent” which denotes one class electrical network reduction. The idea of this method is that one section of the power stays the same which should be analyzed in detail. That is called internal system. In addition, all the remaining buses are reduced by the Ward method and collectively called the external system. [5][6][7] But with the development of the power, it will take long time to complete the reduction by using the traditional method, when the network has thousands of buses.

As the purpose of searching an effective way to stabilize the state and analyze the dynamic response, the mesh structure of the network will be a challenge. Because of the grandiosity of the structure, it is difficult to figure out about the correlation between the different elements in the power network. And the different grad of network as well as its mesh structure will lead to a long calculation process by using the traditional method and a complex result. Hence, one of the most significant current discussions in legal and the moral philosophy is the reduction of the network through a dynamic equivalent model into a transfer function. Through the system identification a dynamic equivalent transfer function can be achieved by the usage of the input and output data from the network. With the different identification methods, the resulted equivalent models are not the same. One of them can accurately represent the original dynamic state and the others are inaccurate. With a serious selection of a validated model, the problem for the efficiency and accuracy of the simulation can be solved.

This paper, on the one hand, presents a range of different system identification methods. On the other hand, it displays the conditions for the use of them, which are considerable before the utilization. The major target of this study is to be committed to find a suitable dynamic equivalent model for the reduced power network and find out the fitted condition for the later application.

II. SYSTEM IDENTIFICATION

The process of the system identification can apply the following flow diagram to solve the difficulty.

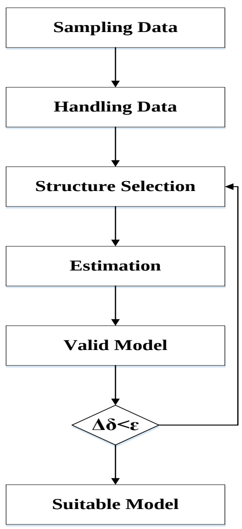

Figure 1: Process of system identification

With using different structure and estimation methods, the results will be different. Some focus on the handling the initial value and some aim to tackle the reduction of the deviation. Therefore, any certain appropriate method should be affirmed on the basis characteristics of system.

A. The structure of model

The most widely used linear regression structure in identification are ARX, ARMAX, OE and BJ. [8]

ARX: This model describes the relational status of output according to input process on the condition that the noise, input and output spectrums have the same characteristic dynamics. The equation is given by

$$\mathrm{y}(q)=\frac{\mathrm{B}}{\mathrm{A}}\mathrm{x}(q)+\frac{1}{\mathrm{A}}\mathrm{e}(q) \tag{1.1}$$

ARMAX: In comparison with ARX, this model has better accuracy, because it contains an extra parameter term in this model structure.

$$\mathrm{y}(q)=\frac{\mathrm{B}}{\mathrm{A}}\mathrm{x}(q)+\frac{\mathrm{C}}{\mathrm{A}}\mathrm{e}(q) \tag{1.2}$$

OE: When the model contains a white noise, this structure can be . In this structure, there is no noise spectrum coefficient.

$$\mathrm{y}(q)=\frac{\mathrm{B}}{\mathrm{F}}\mathrm{x}(q)+\mathrm{e}(q) \tag{1.3}$$

BJ: This model has separate transfer function for the input, output and noise, which is different from the ARX and ARMAX.

$$\mathrm{y}(q)=\frac{\mathrm{B}}{\mathrm{F}}\mathrm{x}(q)+\frac{\mathrm{C}}{\mathrm{D}}\mathrm{e}(q) \tag{1.4}$$

B. The different form of function

The normal descriptions of system are usually completed by the transfer function and state space. When a system is located at SISO form, the transfer function is more proper and the MIMO will be better applied to the state space.[9] The basic form of transfer function is given by

$$\mathrm{G}(s)=\frac{Y(s)}{U(s)}=\frac{\mathcal{L}\{y(t)\}}{\mathcal{L}\{u(t)\}} \tag{1.5}$$

And the state space

$$\sum\nolimits_t \begin{cases} \dot{x}=\mathrm{A}(t)x+\mathrm{B}(t)u,\, x(t)\in R^n, u(t)\in R^q \\ y=\mathrm{C}(t)x+\mathrm{D}(t)u,\, y(t)\in R^q \end{cases} x(t_0)=x_0 \tag{1.6}$$

They can be mutually harmonious to each other.

$$\mathrm{G}(s)=\mathrm{C}(sI-A)^{-1}B+\mathrm{D} \tag{1.7}$$

But when the system contains a timely delay element. A special transfer function can be applied hereof, which is also another kind of transfer function. But this one can better fit the system without any deployment in a better way. [10] It is given by

$$\mathrm{G}(s)=\frac{k_P\left(1+T_z s\right)}{s(1+T_p s)}e^{-T_d s} \tag{1.8}$$

According to different situations, these functions can be applied in various conditions.

C. Method of Parameter Estimation

In this paper, three estimation method will be proposed. Instrument variables, subspace identification and prediction error identification method are widely used.

Instrument variable

Initial value is the premise with regards to an optimal result. Thus, if the initial parameter for the search optimization routine are chosen poorly, its sense is that one cannot ensure the utility value of the global optimal parameter. Since the solution for the initial value is very important, we will put forward a widely known method for finding good initial estimates called instrument variables method. The SISO and LTI transfer function model will be used here.[11]

$$\mathrm{y}(t)=\mathrm{G}(q)\mathrm{u}(t)+\mathrm{H}(q)\mathrm{e}(t) \tag{1.9}$$

$\mathrm{G}(q)$ and $\mathrm{H}(q)$ are rational functions and $\mathrm{e}(t)$ is the white noise with zero mean. As the deterministic part of the function, $\mathrm{G}(q)$ is given by

$$\mathrm{G}(q)=\frac{\mathrm{B}(q)}{\mathrm{A}(q)} \tag{1.10}$$

Then we can rewrite the function as

$$\mathrm{A}(q)\mathrm{y}(t)=\mathrm{B}(q)\mathrm{u}(t)+\mathrm{A}(q)\mathrm{H}(q)\mathrm{e}(t) \tag{1.11}$$

According to the Gauss regression

$$\mathrm{y}(t)=\varphi^{\mathrm{T}}(t)\eta+\upsilon(t) \tag{1.12}$$

The noise can be defined as

$$\upsilon(t)\triangleq\mathrm{A}(q)\mathrm{H}(q)\mathrm{e}(t) \tag{1.13}$$

Then the η should be estimated. So firstly, the $\varsigma(t)$ must be found, which should conform to the following condition

$$\lim_{N\to\infty}\frac{1}{\mathrm{N}}\sum_{t=1}^{N}\varsigma(t)\upsilon(t)=0 \tag{1.14}$$

So it would, asymptotically as $N\to\infty$, hold that

$$\frac{1}{\mathrm{N}}\sum_{t=1}^{N}\varsigma(t)\mathrm{y}(t)=\frac{1}{\mathrm{N}}\sum_{t=1}^{N}\varsigma(t)\varphi^{\mathrm{T}}(t)\eta \tag{1.15}$$

Then an estimation of η could be found via

$$\hat{\eta}_N^{IV} = \arg\min_{n\in R^n} \left\| \frac{1}{N}\sum_{t=1}^{N} \varsigma(t)(y(t)-\varphi^T(t)\eta)\right\|_2^2 \qquad (1.16)$$

The element of $\varsigma(t)$ is referred to as the instrument variables and the ordinary least square estimation of η is retrieved by choosing the $\varsigma(t)=\varphi(t)$.[12]

Subspace identification

This method is presented for the state space model. It is mainly based on linear algebraic techniques, where the projections are used to find the range and null spaces of certain linear mappings which reveal information about the matrices in the state-space model [13].

Subspace algorithms exert themselves to seek to estimate the state-space matrices A,B,C and D of the state space model from input and output data of the system. The main procedure of the subspace identification is given by

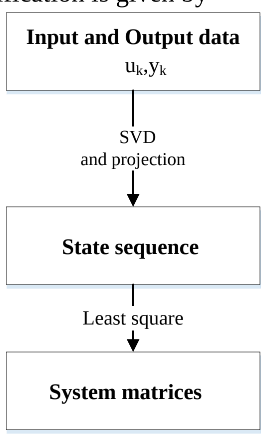

Figure 2 Subspace method

Firstly, the block Hankel matrices are an important concept in subspace identification algorithms.

$$\begin{pmatrix} U_p \\ U_f \end{pmatrix} = \left(\frac{\begin{pmatrix} u_0 & \cdots & u_{j-1} \\ \vdots & \ddots & \vdots \\ u_{i-1} & \cdots & u_{i+j-2} \end{pmatrix}}{\begin{pmatrix} u_i & \cdots & u_{i+j-1} \\ \vdots & \ddots & \vdots \\ u_{i+h-1} & \cdots & u_{i+h+j-2} \end{pmatrix}} \right) = \begin{pmatrix} U_p^{\dagger} \\ U_f^{-} \end{pmatrix} = \left(\frac{\begin{pmatrix} u_0 & \cdots & u_{j-1} \\ \vdots & \ddots & \vdots \\ u_i & \cdots & u_{i+j-1} \end{pmatrix}}{\begin{pmatrix} u_{i+1} & \cdots & u_{i+j} \\ \vdots & \ddots & \vdots \\ u_{i+h-1} & \cdots & u_{i+h+j-2} \end{pmatrix}} \right)$$

U_p represent the past data and U_f means the data in the future. After that, the projection concept is also considerable. The former one is orthogonal projections. For the sake of simplicity, the projection will be conducted in the coordinate of two dimensions.

$$A/B = AB^T(BB^T)^{+}B = A\prod_B \qquad (1.17)$$

Additionally

$$A/B^{\perp} = A(I-\prod_B) = A(I-B^T(BB^T)^{+}B) = A\prod_B^{\perp} \qquad (1.18)$$

These two projections decompose the matric A in to two matrices

$$A = A\prod_B + A\prod_{B^{\perp}} \qquad (1.19)$$

This process can be explained by the figure 3.

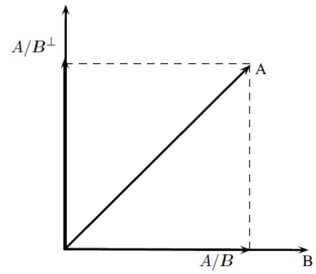

Figure 3 Orthogonal projection of A in two dimensions

The purpose of the orthogonal projection is to remove the noise from the data.

Another projection method is oblique projection. The process is to decompose the row of A as a linear combination of the rows of two non-orthogonal matrices B and C. It will be defined as

$$\begin{pmatrix} B \\ A \end{pmatrix} = LQ = \begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix}\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \qquad (1.20)$$

$$A/B = L_{21}Q_1$$

$$A/B^{\perp} = L_{22}Q_2$$

Matric A along the B and matric B is decomposed on matric C.

$$A\,/_B\,C = A\left(C^T\ B^T\right)\left[\left(\begin{pmatrix} C \\ B \end{pmatrix}\left(C^T\ B^T\right)\right)^{\dagger}\right]C \qquad (1.21)$$

The equation for A is

$$A = A\,/_B\,C + A\,/_C\,B + A/\begin{pmatrix} B \\ C \end{pmatrix} \qquad (1.22)$$

This process can be described by figure 4. It shows an example of an oblique projection of a vector A.

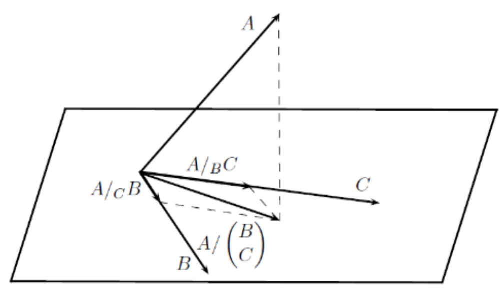

Figure 4 Oblique projection of A

The last step of the subspace identification routine is to estimate the system matrices A, B, C, D. This is completed in a least squares essence. [14]

Prediction Error method

With the help of this method, the prediction error of model plays significant role. This error is the difference between the output of the system as would be predicted by the model and the actual measurement. [15]

From the very beginning of PEM model, firstly we will use a LTI system to illustrate the process.

$$y(t) = G_0(z)u(t) + \upsilon(t) \qquad (1.23)$$

$G_0(z)$ is the transfer function and the disturbance $\upsilon(t)$ can be measurement noise, process disturbance and effects of non-measured input.

$$\upsilon(t) = H_0(z)e(t) \tag{1.24}$$

To find the best prediction, the first will be considered that there is no input

$$y(t) = \upsilon(t) = H(z)e(t) \tag{1.25}$$

As $H(z)$ is stable invers,

$$e(t) = H^{-1}(z)\upsilon(t) \tag{1.26}$$

So the past value of white noise can be constructed.

$$e(t-1) = \upsilon(t-1) + \sum_{k=1}^{\infty} \bar{h}(k)\upsilon(t-1-k) \tag{1.27}$$

Through the transformation

$$e(t) = \upsilon(t) + \left[H^{-1}(z) - 1\right]\upsilon(t) \tag{1.28}$$

Combining the equation gives

$$\upsilon(t) = e(t) + \left[H(z) - 1\right](\upsilon(t) + \left[H^{-1}(z) - 1\right]\upsilon(t)) \tag{1.29}$$

Then we can get

$$\upsilon(t) = e(t) + \left[1 - H^{-1}(z)\right]\upsilon(t) \tag{1.30}$$

The past value of disturbance can be computed from

$$\upsilon(t) = y(t) - G_0(z)u(t) \tag{1.31}$$

And then the prediction can be written

$$\hat{y}(t\,|\,t-1) = H^{-1}(z)G(z)u(t) + \left[1 - H^{-1}(z)\right]y(t) \tag{1.32}$$

Finally, we can get the prediction error

$$\varepsilon(t) = y(t) - \hat{y}(t\,|\,t-1) = H^{-1}(z)\left[y(t) - G(z)u(t)\right] \tag{1.33}$$

The figure 5 shows a schematic for the representation of the calculation of the prediction error. [16][17][18]

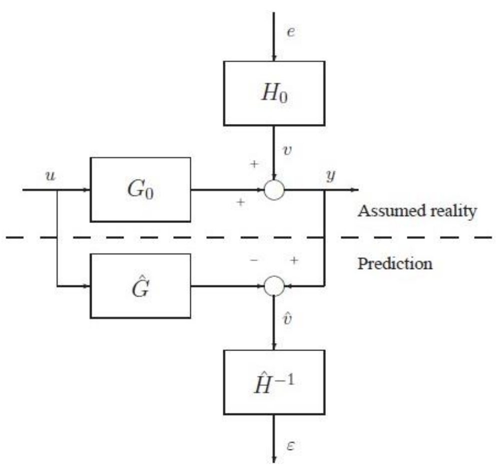

Figure 5 presentation of prediction error

III. CONNECTION OF SIMULATION MODEL

For the sake of finding out the condition of the use for these methods, it makes use of an example to explain for it. In the view of the sampling, the data of a network model in Figure 6 is collected. In this model, the converter will be connected to a 20kV test system that will be connected to a 110kV overlaid fixed electrical grid. The test grid is based on the data of Cigré reference grid for the Europa medium voltage distribution network benchmark, which will be developed and used as the simulation model in this paper. The topology of the proposed test system is in Figure 6, which consists of one radial

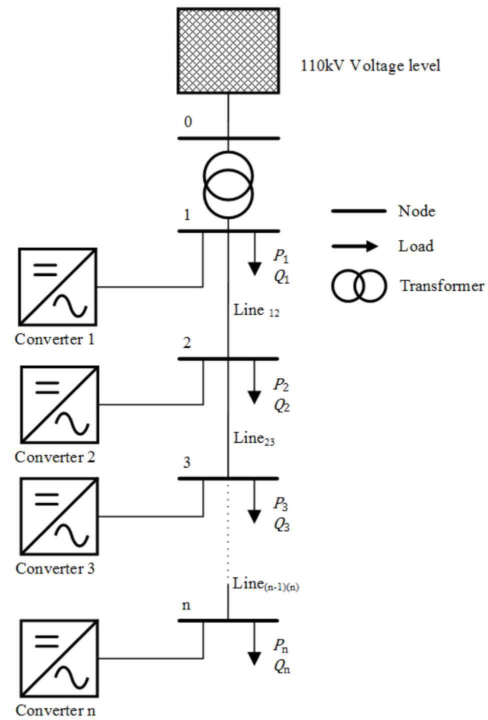

Figure 6 Simulation model

line with the three busses and an overall overhead line length of 7.9km that is allocated to the line 12 and 23 proportionally. The value for the overhead lines will be adapted to the test system in Table 1. [20]

Table 1 Parameter of overhead line in test system

	Resistance [Ω/km]	Inductance [mH/km]	Capacitance [nF/km]
Overhead lines	0.5100	1.1650	10.0968

The signal of input is the voltage from the high voltage side, which is in p.u. size and the output is the power from the middle voltage side. Hereby, the P_2 will be selected as the signal of output, which is given by the figure 7.[1]

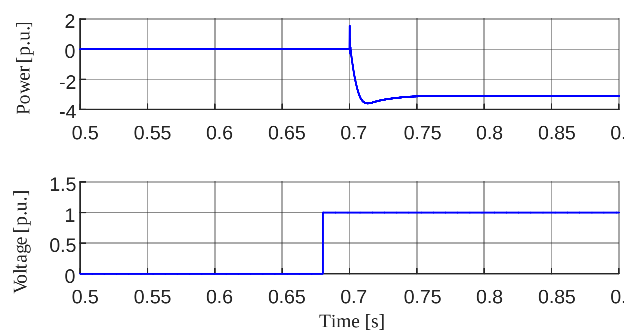

Figure 7 Development of active power with voltage step

Through the IV method, the transfer function is given by

$$G(s) = \frac{-325000s^2 + 87830000s - 9748000000}{s^3 + 247.7s^2 + 39270s + 3136000} \tag{1.34}$$

The development of this function presented dynamic equivalent model is showed in figure 7.
And the result of subspace identification method is

$$\dot{x}=\begin{pmatrix}-21.43 & -401.4 & 378.4\\ 243.4 & -49830 & 76500\\ -146 & 47740 & -73940\end{pmatrix}x+\begin{pmatrix}-0.5911\\ -180.2\\ 185\end{pmatrix}u$$
$$y=\begin{pmatrix}46340 & 6945 & -9478\end{pmatrix}x \qquad (1.35)$$

Subspace estimation is designed for the state space model. Thus, this method is widely used in MIMO system.
Through the PEM, the consequence of calculation can be

$$\dot{x}=\begin{pmatrix}-20.31 & -384.9 & 387.8\\ 248.5 & -49770 & 76500\\ -140.7 & 47810 & -73900\end{pmatrix}x+\begin{pmatrix}0.5328\\ -52.43\\ 61.34\end{pmatrix}u \qquad (1.36)$$
$$y=\begin{pmatrix}46340 & 6932 & -9476\end{pmatrix}x$$

Process model is a special method that can better simulate the system with delay, proportion-integral, because in the structure of this model in (1.8) it has the K_p, K_I and the $e^{-T_d s}$ part. Thus, by the using of it, only the parameters need to be estimated, the structure is regular. Then through the process model, it is given by

$$G(s)=\frac{-3119.2*(1-0.006813s)}{(1+0.0000001s)(1+0.0000004s)(1+0.01471s)}e^{0.003s} \qquad (1.37)$$

From the (1.34) to (1.37) there are four functions presented. Each function represents a dynamic equivalent model. In comparison with the original model, the degree of the coincidence of these methods can be showed in the figure7.

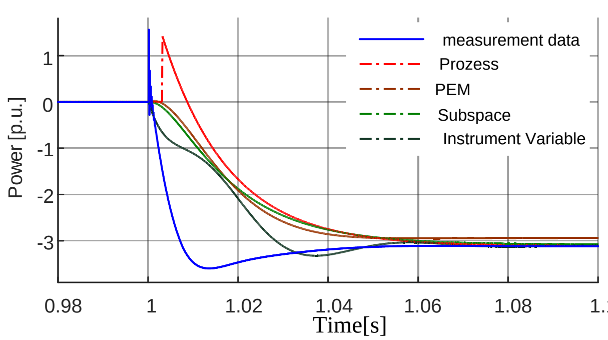

Figure 8 the results of active power by the use of different methods

IV. Discussion and Conclusion

The aim of this research project is to find a proper method to get a dynamic equivalent model to reduce the power network, which is a meaningful way for the research on the stabilizing of the power system. Due to the system identification, the mesh power network can be replaced by an equivalent model in a transfer function or state space, which can be easily used to analyze the dynamic characteristic. On the basis of the consequence of the simulation, factors for the use of these methods can be in accord with the character of them.

It can be concluded that the transfer function form is suitable for finding a dynamic equivalent model for the reduction of power network. By using it, the accuracy and efficiency of the result is reasonable. Normally, the MIMO system can use the state space form, which is not suitable for the single transfer function any more. Then the estimation methods should be affirmed for the confirming system parameter. When the initial value play an important role in the dynamic equivalent model, the IV estimation method should be taken into account. This method lay emphasis on estimation of confirming a precise initial value. The PEM method is used for the prediction of any error. Subspace method is designed for the state space form. Therefore, it can be used in the MIMO system. The process model is usually used in the system with loop control or with the delay and proportional-integral part. From the result in figure 7, the IV method and the process model are appropriate ones for the normal reduction of network, because they are more precise in the dynamic response. Subspace method and PEM method can't be suitable, because the deviation of dynamic state in comparison with the original curve is worse than the above two methods. The following table shows various conditions for the use of these methods and precise degree through the result from the simulation.

Table 2 the comparison of different estimation methods

	Condition for use	Application in the reduction network
IV	Precise initial value	High precise in dynamic state
Subspace	MIMO System	Low precise in dynamic state and high precise in steady state
PEM	Error prediction und better for MIMO System	Low precise in both dynamic and steady state
Process model	System with PI control and delay	High precise in dynamic state

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